Dual Dig Level I (2009)

Unless otherwise stated or implied, assume that all graphs are in the usual xy-plane.

1. The quotient of 2 numbers is equivalent to $\frac{7}{4}$. If the numerator and the denominator are each increased by 20, the quotient becomes equal to $\frac{11}{8}$. Find the original numbers.

2. If $\frac{A}{B}+\frac{4}{3}+\frac{9}{2}=\left(\frac{A}{B}\right)\left(\frac{4}{3}\right)\left(\frac{9}{2}\right)$, find $\frac{A}{B}$ in lowest terms.

3. A line with slope 2 intersects a line with slope 6 at the point $(40,30)$. What is the distance between the x-intercepts of the 2 lines?

4. Solve the system: $\begin{cases} x-2y+3z=9 \\ 3y-x=-4 \\ 2x-5y+5z=17 \end{cases}$

5. A positive real decimal number has the property that, when its decimal point is moved four places to the right, the result is four times the reciprocal of the original number. Find the original decimal number.

6. A bathroom scale is set too high. When Tweedledum stands on the scale, it reads 180 pounds. When Tweedledee (alone) stands on the scale, it reads 240 pounds. When they both stand on the scale, the scale reads 400 pounds. How many pounds too high is the scale set?

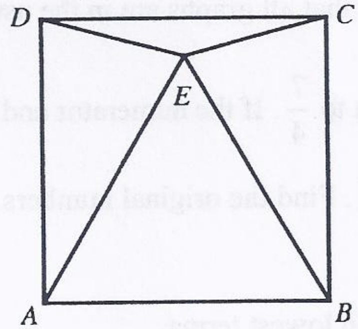
7. $ABCD$ and $DCFE$ are coplanar rectangles with $AB=4$, $AC=5$, and $BC=CF$. (Point B and Point F are distinct points.) What is the exact length of $\overline{AF}$ in simplest form?

8. Find the values of three nonzero distinct digits, "A," "B," and "C," so that the square of the number formed by BC (that is: B in the tens place and C in the ones place) is equal to the number ABC.

9. For a certain two digit number, the tens digit is twice the units digit. If 36 is subtracted from the original number, the digits will be interchanged. Find the original number.

10. If n is a positive integer, then $n!$, called "n factorial," is the product of the integers from 1 through n. Give the digit in the tens place of the integer whose value is $1!+2!+3!+4!+\ldots+2009!$, which is the sum of the factorials of the first 2009 positive integers.

11. Point E is a point inside the square $ABCD$ such that triangle EAB is an equilateral triangle. Find the measure of angle CED.



12. Find the product of: $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\cdots\left(1-\frac{1}{10^2}\right)$, and simplify.

13. A new mathematical operation is formed and defined as: $a \otimes b = (2a+b)^2$.

Calculate: $\sqrt{(3 \otimes 2) \otimes (2 \otimes 3)}$.

14. Assuming that all variables represent positive real numbers, and $g \neq a+b$, solve for x:
$ax - by = gx - bx + 5$.

15. The point $A(5,-2)$ is reflected on a perpendicular over the line $x - 3y = 1$. What are the coordinates of the reflected point A'?

16. The graph of $y = ax^3 + bx^2 + cx + d$ includes the points $(-1,0)$, $(1,0)$, and $(0,3)$. Assume that a, b, c, and d represent real constants. What must be the value of b?

17. An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle is ten square meters, what is the area of the hexagon? Hint: Draw the triangle with line segments connecting the midpoints of the sides.

18. Which of the following three numbers is greatest in value: $\sqrt{2}$, $\sqrt[3]{3}$, or $\sqrt[6]{6}$?

19. Solve for x: $2x + 3 \leq |x+5| \leq 6x - 9$. Consider only real values for x.

20. Al, Betty, Carl, and Dina play a series of one-on-one basketball games, and their P.E. teacher hands out money based on the results. Each player plays a game with each other player exactly once. For each game, if there is a winner, then the winner wins \$2 and the loser gets \$0; in the event of a tie, each player gets \$1. After all the games have been played, Al has won \$3, Betty has won \$4, and Carl has won \$1. How much money has Dina won?